ΠΑΡΑΤΗΡΗΣΕΙΣ ΚΑΙ ΕΠΑΝΑΛΗΠΤΙΚΕΣ ΑΣΚΗΣΕΙΣ - ΛΥΣΕΙΣ

§ 7.8 και 7.9 Βιβλίο Α Γυμνασίου : Δυνάμεις Ρητών

ΠΑΡΑΤΗΡΗΣΕΙΣ

Δεν υπάρχουν ιδιότητες για πρόσθεση και αφαίρεση δυνάμεων	όταν ν άρτιος $\alpha^{\nu} = (-\alpha)^{\nu} \neq -\alpha^{\nu}$	$\frac{1}{a^1} = a^{-1}$	$1 = \frac{a^{\nu}}{a^{\nu}} = a^{\nu-\nu} = a^0$

ΑΣΚΗΣΕΙΣ

Α. Να υπολογίσετε τις δυνάμεις :

1) $-4^2 = -(4\cdot 4) = -16$	**2)** $(-4)^2 = (-4)\cdot(-4) = +16$	**3)** $-4^3 = -(4\cdot 4\cdot 4) = -64$	**4)** $(-4)^3 = (-4)\cdot(-4)\cdot(-4) = -64$
5) $3^{-3} = \frac{1}{3^3} = \frac{1}{27}$	**6)** $(-3)^{-3} = \frac{1}{(-3)^3} = \frac{1}{(-3)(-3)(-3)} = \frac{1}{-27}$	**7)** $(-84)^0 = 1$	**8)** $-84^0 = -1$
9) $\left[(-3)^{-2}\right]^2 = \left[\frac{1}{(-3)^2}\right]^2 = \left(\frac{1}{9}\right)^2 = \frac{1^2}{9^2} = \frac{1}{81}$		**10)** $5^2\cdot 5^{-2} = 5^2\cdot\frac{1}{5^2} = \frac{5^2}{5^2} = 1$	
11) $\frac{12^{-3}}{4^{-3}} = \left(\frac{12}{4}\right)^{-3} = 3^{-3} = \left(\frac{1}{3}\right)^3 = \frac{1^3}{3^3} = \frac{1}{27}$		**12)** $\left(\left(\frac{1}{2}\right)^2\right)^3 = \left(\frac{1^2}{2^2}\right)^3 = \left(\frac{1}{4}\right)^3 = \frac{1^3}{4^3} = \frac{1}{64}$	

Β. Να υπολογίσετε την τιμή των παραστάσεων :

1) $\frac{3\cdot 4^5}{3^4\cdot 4^2} = \frac{3^1}{3^4}\cdot\frac{4^5}{4^2} = 3^{-3}\cdot 4^3 = \frac{1}{3^3}\cdot 4^3 = \frac{64}{27}$	**2)** $\frac{2^2\cdot 5^5}{5^4\cdot 2^7} = \frac{2^2}{2^7}\cdot\frac{5^5}{5^4} = 2^{-5}\cdot 5^1 = \frac{1}{2^5}\cdot 5 = \frac{5}{32}$
3) $\left(\frac{3}{4}\right)^{-2}\cdot\frac{(-3)^{-2}}{4} = \left(\frac{4}{3}\right)^2\cdot\frac{1}{(-3)^2\cdot 4} = \frac{4^2}{3^2}\cdot\frac{1}{9\cdot 4} = \frac{16}{9}\cdot\frac{1}{36} = \frac{16}{324} = \frac{4}{81}$	**4)** $(0,25)^6\cdot(-4)^6 = [0,25\cdot(-4)]^6 = (-1)^6 = 1$
5) $\frac{\left(2^3\right)^4}{2^{10}} = \frac{2^{3\cdot 4}}{2^{10}} = \frac{2^{12}}{2^{10}} = 2^{12-10} = 2^2 = 4$	
6) $\frac{(-82)^3}{41^3} - \frac{64^4}{(-32)^4} = \left(\frac{-82}{41}\right)^3 - \left(\frac{64}{-32}\right)^4 = (-2)^3 - (-2)^4 = (-8) - (+16) = -8 - 16 = -24$	
7) $\left(\frac{-1}{2006}\right)^{2007}\cdot 2006^{2007} = \frac{(-1)^{2007}}{2006^{2007}}\bullet 2006^{2007} = \frac{(-1)^{2007}\bullet 2006^{2007}}{2006^{2007}} = \frac{[(-1)\bullet 2006]^{2007}}{2006^{2007}} = \left(\frac{-2006}{2006}\right)^{2007} = (-1)^{2007} = -1$	
8) $\left(\frac{-1}{2007}\right)^{2006}\cdot 2007^{2006} = \frac{(-1)^{2006}}{2007^{2006}}\bullet 2007^{2006} = \frac{(-1)^{2006}\bullet 2007^{2006}}{2007^{2006}} = \frac{[(-1)\bullet 2007]^{2006}}{2007^{2006}} = \left(\frac{-2007}{2007}\right)^{2006} = (-1)^{2006} = 1$	
9) $(-2)^3\bullet 2 - 3^2 + (-2)^4 : 16 + [-1-(-1)^9\bullet 8] = -8\cdot 2 - 9 + \frac{(-2)^4}{16} + [-1-(-1)\cdot 8] =$ $= -16 - 9 + \frac{16}{16} + [-1-(-8)] = -25 + 1 + (-1+8) = -24 + 7 = -17$	
10) $[(-21,5)^2\bullet(21,5)^{-2}]^5 - \frac{12^{-3}}{6^{-3}} + 4\bullet 2^{-2} = [(21,5)^2\bullet(21,5)^{-2}]^5 - \left(\frac{12}{6}\right)^{-3} + 4\cdot\frac{1}{2^2} =$ $= [(21,5)^{2-2}]^5 - 2^{-3} + \frac{4}{4} = [(21,5)^0]^5 - \frac{1}{2^3} + 1 = 1^5 - \frac{1}{8} + 1 = 2 - \frac{1}{8} = \frac{16}{8} - \frac{1}{8} = \frac{15}{8}$	
11) $\frac{\left(-\frac{3}{2}\right)^4\cdot 2^4 - 3^4 + 10}{\left[1-(-1)^{2015}\right]^0} = \frac{\left(-\frac{3}{2}\cdot 2\right)^4 - 81 + 10}{1} = (-3)^4 - 81 + 10 = 81 - 81 + 10 = 10$	
12) $\frac{\left[(-2)^2 + (-1)^2\right]^2}{5} + 5 = \frac{(4+1)^2}{5^1} + 5 = \frac{5^2}{5^1} + 5 = 5^{2-1} + 5 = 5 + 5 = 10$	

Γ. Αν ο ν είναι άρτιος αριθμός, να βρεθεί η τιμή της παράστασης : $A=4\cdot(-1)^{\nu}+3\cdot\frac{(-1)^{2\nu+1}}{5}-7\cdot\frac{(-1)^{3\nu}}{5}$

$$A=4\cdot(-1)^{\nu}+3\cdot\frac{(-1)^{2\nu+1}}{5}-7\cdot\frac{(-1)^{3\nu}}{5}=4\cdot1+3\cdot\frac{-1}{5}-7\cdot\frac{1}{5}=4+\frac{-3}{5}-\frac{7}{5}=4-\frac{3}{5}-\frac{7}{5}=4-\frac{10}{5}=4-2=2$$

Δ. Αν ο ν είναι περιττός αριθμός, να βρεθεί η τιμή της παράστασης : $A=4\cdot(-1)^{\nu}+2\cdot\frac{(-1)^{2\nu+1}}{5}-7\cdot\frac{(-1)^{3\nu}}{5}$

$$A=4\cdot(-1)^{\nu}+2\cdot\frac{(-1)^{2\nu+1}}{5}-7\cdot\frac{(-1)^{3\nu}}{5}=4\cdot(-1)+2\cdot\frac{(-1)}{5}-7\cdot\frac{-1}{5}=-4-\frac{2}{5}+\frac{7}{5}=-4+\frac{5}{5}=-4+1=-3$$